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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Thursday 15 May 2014 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 14]*

The random variable X has probability distribution $\text{Po}(8)$.

(a) (i) Find $\text{P}(X = 6)$.

(ii) Find $\text{P}(X = 6 \mid 5 \le X \le 8)$. *[5]*

(b) $\bar{X}$ denotes the sample mean of $n > 1$ independent observations from X.

(i) Write down $\text{E}\left(\bar{X}\right)$ and $\text{Var}\left(\bar{X}\right)$.

(ii) Hence, give a reason why $\bar{X}$ is not a Poisson distribution. *[3]*

(c) A random sample of 40 observations is taken from the distribution for X.

(i) Find $\text{P}\left(7.1 < \bar{X} < 8.5\right)$.

(ii) Given that $\text{P}\left(|\bar{X} - 8| \le k\right) = 0.95$, find the value of k. *[6]*

2. *[Maximum mark: 16]*

The following table gives the average yield of olives per tree, in kg, and the rainfall, in cm, for nine separate regions of Greece. You may assume that these data are a random sample from a bivariate normal distribution, with correlation coefficient ρ.

Rainfall (x)	11	10	15	13	7	18	22	20	28
Yield (y)	56	53	67	61	54	78	86	88	78

A scientist wishes to use these data to determine whether there is a positive correlation between rainfall and yield.

(a) State suitable hypotheses. *[2]*

(b) Determine the product moment correlation coefficient for these data. *[2]*

(c) Determine the associated p-value and comment on this value in the context of the question. *[2]*

(d) Find the equation of the regression line of y on x. *[2]*

(e) Hence, estimate the yield per tree in a tenth region where the rainfall was 19 cm. *[2]*

(f) Determine the angle between the regression line of y on x and that of x on y. Give your answer to the nearest degree. *[6]*

3. *[Maximum mark: 14]*

(a) Consider the random variable X for which $\mathrm{E}(X) = a\lambda + b$, where a and b are constants and λ is a parameter.

Show that $\dfrac{X-b}{a}$ is an unbiased estimator for λ. *[3]*

(b) The continuous random variable Y has probability density function

$$f(y) = \begin{cases} \dfrac{2}{9}(3 + y - \lambda), & \text{for } \lambda - 3 \le y \le \lambda \\ 0, & \text{otherwise} \end{cases}$$

where λ is a parameter.

(i) Verify that $f(y)$ is a probability density function for all values of λ.

(ii) Determine $\mathrm{E}(Y)$.

(iii) Write down an unbiased estimator for λ. *[11]*

4. *[Maximum mark: 16]*

Consider the random variable $X \sim \text{Geo}(p)$.

(a) State $\text{P}(X < 4)$. *[2]*

(b) Show that the probability generating function for X is given by $G_X(t) = \frac{pt}{1-qt}$, where $q = 1 - p$. *[3]*

Let the random variable $Y = 2X$.

(c) (i) Show that the probability generating function for Y is given by $G_Y(t) = G_X\left(t^2\right)$.

(ii) By considering $G'_Y(1)$, show that $\text{E}(Y) = 2\text{E}(X)$. *[6]*

Let the random variable $W = 2X + 1$.

(d) (i) Find the probability generating function for W in terms of the probability generating function of Y.

(ii) Hence, show that $\text{E}(W) = 2\text{E}(X) + 1$. *[5]*